$$\omega\tau \equiv \frac{(\omega_e\tau_e)^*}{(\omega\tau)_1^*} = \frac{B}{\rho} \tag{7}$$

Since it was assumed that segmented electrodes were used, such that j^* was in the y^* direction only, an axial electric field E_x^* is induced which can be found from[4]

$$E_x^* = (\omega_e\tau_e)^* j^*/\sigma^*$$

where ion slip is again neglected In nondimensional variables,

$$E_x \equiv \left[\frac{E_x}{u_1B_1(\omega\tau)_1}\right]^* = \frac{Bj}{\rho\sigma}$$

$$= \frac{Bj}{\rho} \quad \text{for } \sigma = 1 \tag{8}$$

The net potential difference between $x^* = 0$ and x^* is

$$\Phi^* = -\int_0^{x^*} E_x^* dx^*$$

or

$$\Phi \equiv \frac{\Phi^*}{[u_1B_1(\omega\tau)_1L]^*} = -\int_0^x E_x dx =$$

$$p_1 \ln\left(\frac{1}{\rho}\right) - \left(\frac{u^2-1}{2}\right) \tag{9}$$

The upstream and downstream ends of the accelerator must be electrically insulated from each other in order to prevent a current flow in the x direction

Equations (4–9) define the present solution There are three free parameters in the nondimensional equations: E_{y1}, n, and p_1 If the inlet flow conditions to the accelerator are specified, p_1 is defined, and only the parameters $E_{y,1}$ and n remain The length x, required to achieve a velocity u, decreases as E_{y1} and n increase However, it is desirable to have the values of n and $(E_{y1} - 1)$ as near to zero as possible without incurring excessive accelerator length The case $n = 0$ corresponds to uniform energy addition per unit x, and all portions of the accelerator are equally important As n increases, the downstream portions of the accelerator add proportionately more energy (and may do so inefficiently) Values of E_{y1} near 1 correspond to relatively little joule heating As E_{y1} increases, the fraction of the input energy which goes into joule heating also increases, and the downstream channel area increases (in order to keep the enthalpy constant) This causes lower p,ρ (which limits the tunnel altitude simulation) and higher $\omega\tau$ (which may lead to ion slip) at the downstream stations Numerical evaluation of Eqs (4–9) for various values of n and E_{y1} will indicate the appropriate values for reasonable accelerator length, $\omega\tau$, and tunnel altitude simulation

The present solution requires that E_y and B vary with x, but this should not cause difficulty in an experimental facility The use of segmented electrodes permits E_y to be varied at will Similarly, the spacing between the sides of an air core magnet can be varied to provide the correct variation of B with x Equations (4–9) have been used in a preliminary design of a pulsed-type hypervelocity wind tunnel This design will be reported in a later publication

Appendix

The present solution can be readily generalized to the case where h and jE_yA are specified functions of u and A is a specified function of x,u The equation of state [Eq (2e)] and the scalar conductivity law [Eq (2f)] can also be arbitrarily specified The relation between x and u becomes [from Eq (3)]

$$x = \int_1^u \left(\frac{dh/du + u}{jE_yA}\right) du \tag{A1}$$

Equation (A1) can be integrated in closed form for numerous functional forms of $h(u)$ and $jE_yA(u)$ The remaining dependent variables are found from $\rho = 1/uA$; $p = p(\rho,h)$; $\sigma = \sigma(p,h)$, $j^2 = \sigma(\rho u dh/du - u dp/du) du/dx$; $B = (\rho u + dp/du)(du/dx)/j$, $E_y = uB + j/\sigma$, $E_x = Bj/\rho\sigma$, and

$$-\Phi = \int_1^u \frac{udu}{\sigma} + \int_{p_1}^p \frac{dp}{\rho\sigma}$$

The choice of the functional forms of h, jE_yA, and A should be such that the foregoing expressions for j^2 and B are satisfied at $x = 0$

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Optimum Geometric Factors for Semicircular Fins in Radiation-Cooled Nozzles

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Radiation heat transfer may enhance the cooling of rocket surfaces The use of semicircular fins attached to a cylindrical rocket body was investigated The fins are comprised of an equal number of concave and convex sections with unequal radii Equations are derived for the fins, radiation geometric factors, and the efficiency of finned surfaces The equations are programmed on the IBM 704 computer, and the results of the calculations are plotted The efficiency of the finned surfaces considered was significantly improved when the ratio of convex fin radius to concave fin radius was chosen as greater than one

Nomenclature

A	= fin area, ft²
A, A_x	= total surface of concave and convex fin area, respectively, ft²
a,b,c,d,e,f	= angle, deg
E_f	= effective (approximate) fin efficiency, dimensionless
F	= geometric factor, dimensionless
F_c, F_x	= geometric factor for concave and convex fin, respectively, dimensionless
M_1,M_2,M_3,M_4	= slopes of tangents 1, 2, 3, and 4, respectively, dimensionless
Q	= heat dissipated by radiation, Btu/sec
R_p	= outer radius of nonfinned cylinder, ft

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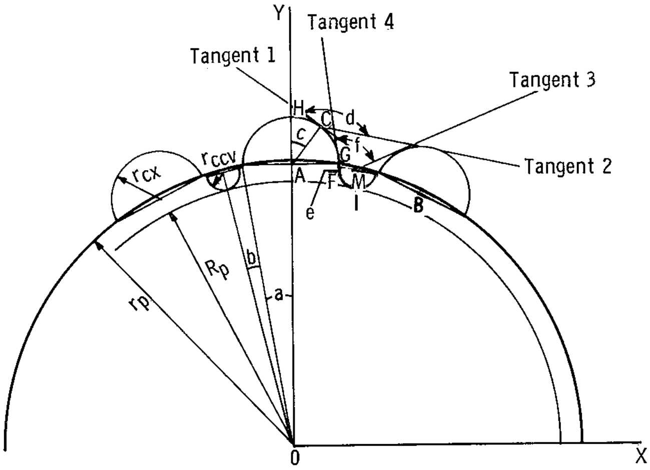

Fig 1 Cross section of finned, radiation-cooled nozzle

r_p = radius of circle passing through junction points of concave and convex fins, ft [defined in Eq (11)]
$r_{\ v}, r_{\ x}$ = radius of concave and convex fins, respectively, ft
T_f, T_s = fin and sink temperatures, respectively, °R
ϵ = emissivity, dimensionless
σ = Stefan Boltzmann constant, Btu/ft²/sec/ R⁴

THE presence of severe thermal conditions inside many modern rockets requires the use of efficient devices for heat dissipation One mode of heat transfer, i e , radiation, can be greatly improved by fins attached to the rocket's outer surface

This note presents an approximate analysis of certain finned surfaces The fins studied are comprised of an equal number of semicircular concave and convex surfaces A sketch of the cross section of such finned surfaces is shown in Fig 1

$$M_2 = \frac{-(X_C - X_B)(Y_C - Y_B) \pm r_{cx}[(X_C - X_B)^2 + (Y_C - Y_B)^2 - r_{\ x}^2]^{1/2}}{r_{\ x}^2 - (X_C - X_B)^2} \tag{6}$$

Analytical studies of finned surfaces result in complex and unwieldy equations [1] Certain simplifying assumptions are made in the present investigation to facilitate calculations They are 1) the surface temperature of the fins is constant at all points of the fins, and 2) the fins are attached to a cylindrical body of constant radius

The heat radiated from a section of a finned surface may be divided in two portions: 1) heat absorbed by the atmosphere, and 2) heat intercepted by adjoining sections of the surface

The radiation geometric factor F is the fraction of the radiated heat which is absorbed by the atmosphere For a nonfinned cylindrical surface, F usually equals one

The geometric factor F is calculated by dividing the fin in sections and obtaining an average geometric factor for each section Since each section is taken quite small, the geometric factor may be considered constant at all points of that section

The over-all geometric factor F is calculated by combining the geometric factors of each section as follows

Let A_1, A_2, A_n be the area of each fin section Let A be the total fin area Then, as the fin temperature is assumed constant at all points of the fin,

$$AF = \sum_{n=1}^{n=n} A_n F_n \tag{1}$$

The heat dissipated by radiation is

$$Q = \epsilon\sigma\, AF(T_f^4 - T_s^4) \tag{2}$$

Equations for the Geometric Factor

In Fig 1 the cross section of a finned surface is divided into two sections: a plain section of radius R_p and center O and a finned section A coordinate system of center O bisecting a convex fin is chosen Let A and B be the centers of circular convex fins separated by a concave fin of center M Note that the finned surface is designated concave or convex with respect to an observer outside of the nozzle Because of the symmetry of the finned surface we calculate only the geometric factor for two adjoining quarter circles, one convex and the other concave The two quarter-circles studied are HCG and GFI

Part 1: Convex fin

Let r_p be the radius of the circle of center O passing through the junction points of convex and concave fins Let C be an arbitrary point on the surface of the convex half-fin and c the angle between line AC and the ordinate We denote by tangent 1 the tangent to the convex fin at the point C Tangent 2 is the line passing through point C and tangent to the fin of center B The angle d between tangent 1 and tangent 2 determines the geometric factor at point C

$$F = (\pi - d)/\pi \tag{3}$$

If M_1 and M_2 are the slopes of tangent 1 and tangent 2, respectively, then

$$d = \tan^{-1}\left(\frac{M_1 - M_2}{1 + M_1 M_2}\right) \tag{4}$$

From geometric considerations, the slopes M_1 and M_2 may be readily calculated Let X_B, Y_B and X_C, Y_C be the coordinates of the points B and C Then we have

$$M_1 = -\tan c \tag{5}$$

The coordinates of the points B and C are

$$X_C = r_{\ x} \sin c \tag{7}$$

$$Y_C = (r_{\ p}^2 - r_{\ x}^2)^{1/2} + r_{\ x} \cos c \tag{8}$$

$$X_B = r_p \sin 2\left[\sin^{-1}\left(\frac{r_{cx}}{r_p}\right) + \sin^{-1}\left(\frac{r_{ccv}}{r_p}\right)\right] \tag{9}$$

$$Y_B = r_p \cos 2\left[\sin^{-1}\left(\frac{r_{cx}}{r_p}\right) + \sin^{-1}\left(\frac{r_{ccv}}{r_p}\right)\right] \tag{10}$$

$$r_p = [(R_p + r\ \ )^2 + r\ \ ^2]^{1/2} \tag{11}$$

Note that Eq (6) provides two values for M_2 The greater value is the correct one to use here

Part 2: Concave fin

Let F be any point on the surface of the concave half-fin and e the angle between lines MF and MG Tangent 3 is the line passing through F and tangent to the fin of center B Tangent 4 is the line passing through F and tangent to the fin of center A The angle f between tangent 3 and tangent 4 determines the geometric factor at the point F

$$F = f/\pi \tag{12}$$

If M_3 and M_4 are, respectively, the slopes of tangent 3 and tangent 4, then

$$f = \tan^{-1}\left(\frac{M_4 - M_3}{1 + M_3 M_4}\right) \tag{13}$$

Let X_A, Y_A and X_F, Y_F be the coordinates of the points A and F For the evaluation of M_3, two cases are distinguished

Case 1: $X_F > X_G$

$$M_3 = \frac{-X_F(Y_F - Y_A) \pm r_{cx}[X_F^2 + (Y_F - Y_A)^2 - r_{cx}^2]^{1/2}}{r_{cx}^2 - X_F^2} \quad (14)$$

The coordinates of points A and F are

$$X_A = 0 \quad (15)$$

$$Y_A = (r_p^2 - r_{cx}^2)^{1/2} \quad (16)$$

$$X_F = (r_p^2 - r_{cv}^2)^{1/2} \sin(a + b) - r_{ccv} \cos(e - a - b) \quad (17)$$

$$Y_F = (r_p^2 - r_{ccv}^2)^{1/2} \cos(a + b) + r_{ccv} \sin(a + b - e) \quad (18)$$

The angle $(a + b)$ corresponds to the section of the circle of radius R_p taken up by one-half concave and one-half convex fins Analytically, $(a + b)$ is

$$a + b = \sin^{-1}(r_{cx}/r_p) + \sin^{-1}(r_{ccv}/r_p)$$

Note that Eq (14) provides two values of M_3 The smaller one is the correct one to use

Case 2: $X_F < X_G$

$$M_3 = (Y_G - Y_F)/(X_G - X_F) \quad (19)$$

M_4 is calculated from the equation

$$-(X_F - X_B)(Y_F - Y_B) \pm M_4 = \frac{r_{cx}[(Y_F - Y_B)^2 + (X_F - X_B)^2 - r_{cx}^2]^{1/2}}{r_x^2 - (X_F - X_B)^2} \quad (20)$$

Note that Eq (20) provides two values of M_4 The greater value is the correct one to use

Fin Effectiveness

The following equation is used as an approximate measure of the effectiveness of a finned surface:

E_f = effective (approximate) fin efficiency =

$$\frac{A_{ccv}F_{ccv} + A_{cx}F_{cx}}{A_p} \quad (21)$$

where A_p is the outer surface of nonfinned cylinder, ft^2 After some manipulations, the following final equation is obtained:

$$E_f = \frac{r_{cx}F_{cx} + r_{ccv}F_{ccv}}{R_p(a + b)/90} \quad (22)$$

where $a = \sin^{-1}(r_x/r_p)$ and $b = \sin^{-1}(r_v/r_p)$

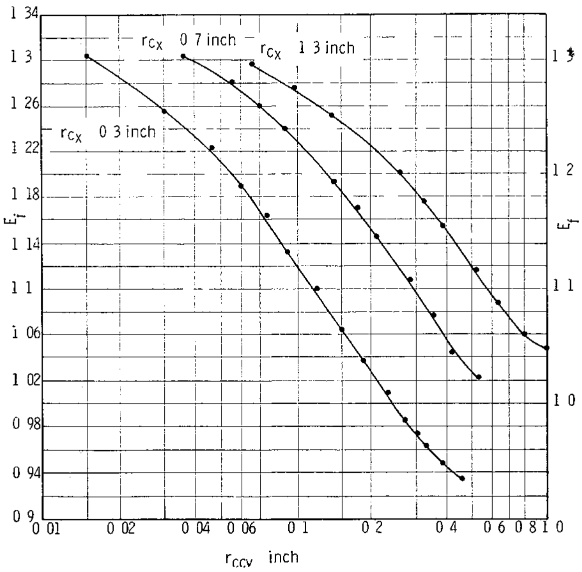

Fig 2 Fin efficiency as function of convex and concave radii (R_p = 5 in)

Calculations

The equations shown were programmed for the IBM 704 computer, and the fin efficiency for various values of R_p, r_x, and r was calculated The results of the computations are shown in Fig 2 It is worth noting that although Fig 2 is based on a radius R_p of 5 in , it is applicable to any radius R_p in view of the following fact: if $[(r_x)_1/(R_p)_1] = [(r_x)_2/(R_p)_2]$ (where the subscripts 1 and 2 refer to different finned surfaces) and $[(r_x)_1/(r_c)_1] = [(r_x)_2/(r_c)_2]$, then $(E_f)_1 = (E_f)_2$

Conclusions

An increase in the ratio of convex to concave fin radii always results in an improved fin efficiency

If two finned surfaces, denoted by the subscripts 1 and 2, are such that

$$[(r_{cx})_1/(r\ )_1] = [(r_x)_2/(r\ ]_2]$$

and

$$[(r_x)_1/(R_p)_1] = [(r_x)_2/(R_p)_2]$$

then $(E_f)_1 = (E_f)_2$

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Integral Equations for Viscous Hypersonic Nozzle Flow

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SCHLICHTING[1] has shown that in incompressible diffusers the integral equations for the combined core and boundary-layer flows have the same form as the usual boundary-layer integral equations In the present note it is shown that this result remains valid in conical hypersonic nozzles in which the thick hypersonic boundary layers and the core flow interact

Following Schlichting[1] the conservation equations will be simultaneously formulated for both the core and boundary-layer flows Since the boundary layer thickness is of the same order as the nozzle radius, the effect of transverse curvature, which has been discussed by Durand and Potter[2] and by Michel,[3] must be taken into account The coordinate system used is shown in Fig 1 and is the same as that in Refs 2 and 3 In the case of turbulent flow all parameters are temporal averages

The flow is divided into a useful inviscid core and a boundary layer of thickness δ It is assumed that the core flow behaves as a spherical source even when the boundary layer

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